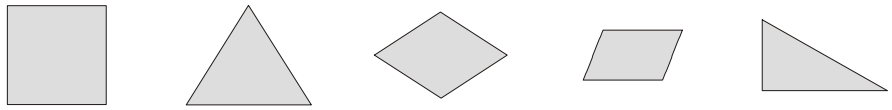
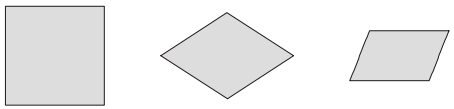
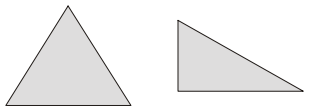
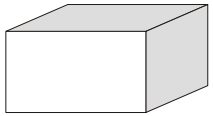
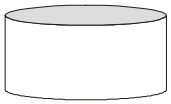
GEOMETRIC AND SPATIAL RELATIONSHIPS

Kindergarten

BIG IDEA (1): Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

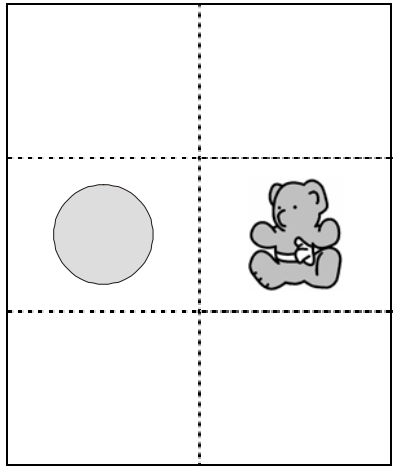
	CONCEPT	EXPECTATION	EXAMPLE
A	Describe and use geometric relationships	Sort 2- and 3-dimensional shapes using physical models (circle, rectangle, triangle, sphere, rectangular prism, cylinder, pyramid)	Give students a collection of shapes, and have them sort the shapes by attributes that are alike. Problem: Use the number of sides each shape below has to sort them into two groups.  Answer: 4 sides  3 sides 

CONCEPT	EXPECTATION	EXAMPLE
		Problem:   rectangular prism cylinder Using the models above, place an R beside each item below that is a rectangular prism, a C beside each item that is a cylinder, and an N beside each item that is neither. 1. cereal box 2. globe 3. can of green beans Answer: 1. R 2. N 3. C

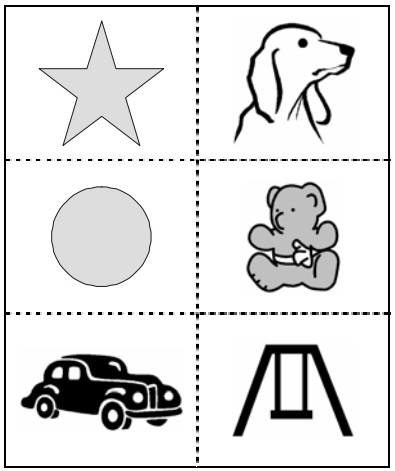
CONCEPT	EXPECTATION	EXAMPLE
		TEACHER NOTES: "Naming shapes is important, but even more important is observing the attributes, or characteristics, of shapes. Characteristics of two-dimensional shapes that children might explore include • the number of sides and vertices (corners), and • how shapes can be put together or taken apart to make other shapes." [1]

[1] National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (pp. 9–10). Reston, VA: Author.

BIG IDEA (2): Specify locations and describe spatial relationships using coordinate geometry and other representational systems

	CONCEPT	EXPECTATION	EXAMPLE
A	Use coordinate systems	Describe, name and interpret relative positions in space (above, below, front, behind)	Have students sit in a semi-circle facing an empty chair. Place a bear or other stuffed animal in front of, behind, over, under, to the right and left of the chair, while saying to the children, "I am putting the bear above the chair, etc." Then call on volunteers, one at a time, to place the bear according to your directions.[2] Problem: Using the grid and pictures below, follow your teacher's directions, and glue each shape on the grid.  

[2] National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (p. 32). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE
		(Teacher's Directions: • Place the dog above the bear. • Place the star above the circle. • Place the swing below the bear. • Place the car below the circle.) Answer:  TEACHER NOTES: "The ability to describe spatial relationships helps us communicate locations and the relative position of two or more objects or persons. Positional words are used to describe one object in relationship to another. Following or giving directions using these words focuses students' attention on positional relationships." [3]

[3] National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (p. 34). Reston, VA: Author.

BIG IDEA (4): Use visualization, spatial reasoning and geometric modeling to solve problems

	CONCEPT	EXPECTATION	EXAMPLE
A	Recognize and draw three-dimensional represen-tations	Recognize geometric shapes in the student's environment (stop sign, number cube, ball)	Teachers can find many opportunities and resources for encouraging students to observe shapes in their environment. For example, teachers can take children on a shape hunt through the school, the playground, or in the nearby neighborhood. *For additional shape-recognition activities, go to "I've Seen That Shape Before" (activities for Pre-K–2) at illuminations.nctm.org.* Problem: Draw an arrow from each picture below to the shape name.  triangle cube circle Answer:  triangle cube circle

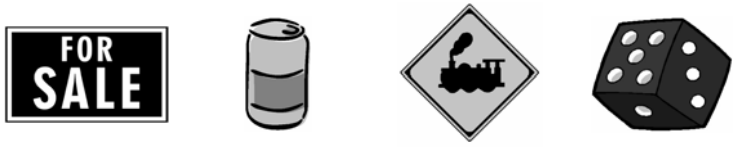
CONCEPT	EXPECTATION	EXAMPLE
		TEACHER NOTES: "Visualization refers to a student's ability to construct a single image or a series of related images, whether the image is of an object or a set of directions. Sometimes visualizations involve imaging what an object looks like from a different point of view, for example, from above rather than from the side. Other times, it involves predicting what an object will look like after it is turned or flipped or what the shadow of an object might look like." [4]

[4] National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (p. 67). Reston, VA: Author.

GEOMETRIC AND SPATIAL RELATIONSHIPS

Grade 1

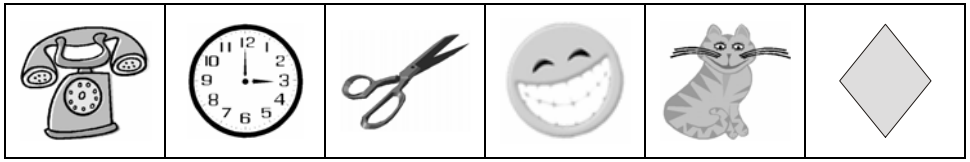
BIG IDEA (1): Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

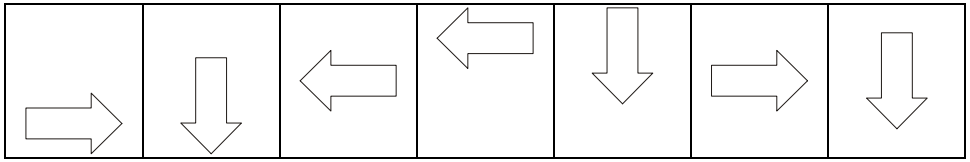
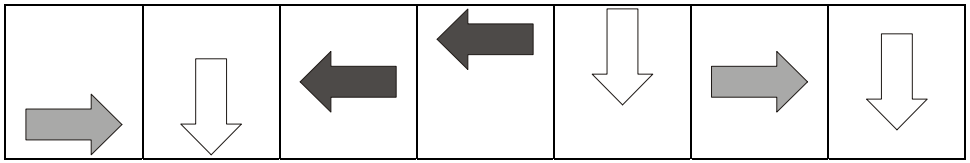
	CONCEPT	EXPECTATION	EXAMPLE
A	Describe and use geometric relationships	Recognize and name 2- and 3-dimensional shapes using physical models (circle, triangle, trapezoid, rectangle, rhombus, sphere, rectangular prism, cylinder, pyramid)	Give students a collection of shapes, and have them sort the shapes by attributes that are alike. Students should be able to identify and name the shapes listed in the grade-level expectation (e.g., circle, triangle, trapezoid). Problem:  A B C D 1. Which picture shows a cylinder? 2. Which picture shows a rectangle? 3. Which picture shows a cube? 4. Which picture shows a rhombus?

CONCEPT	EXPECTATION	EXAMPLE
		Answer: 1. B 2. A 3. D 4. C TEACHER NOTES: "Naming shapes is important, but even more important is observing the attributes, or characteristics, of shapes. Characteristics of two-dimensional shapes that children might explore include • the number of sides and vertices (corners), and • how shapes can be put together or taken apart to make other shapes." Characteristics of three-dimensional figures that children might focus on include the number and shape of the faces, the number of edges and corners, places where the figure is wide or narrow, and what the figure would look like if it were cut apart and opened up.[1]

[1] National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (pp. 9–11). Reston, VA: Author.

BIG IDEA (2): Specify locations and describe spatial relationships using coordinate geometry and other representational systems

	CONCEPT	EXPECTATION	EXAMPLE
A	Use coordinate systems	Describe, name and interpret relative positions in space (left, right)	Distribute the following to each pair of students: • two sheets of four-quadrant grid paper; • two identical sets of 3–5 different geometric shapes (e.g., pattern blocks, Geoblocks, attribute blocks); • barrier (e.g., file folder, tall book) One student places his/her shapes on the grid paper, keeping them hidden from the other student by placing a barrier between the two grid papers. The first student then describes where each shape is located so that the second student can place his/her shapes on the grid paper in the same place. After the first student has given all the directions, and the second student is ready, the barrier is removed, and the two students check to see how well directions were given and followed. They then reverse roles. Problem: Use the strip to answer each question below.  1. Which picture is to the right of the clock? 2. The telephone is to the left of which picture? 3. Which picture is to the left of the rhombus?

CONCEPT	EXPECTATION	EXAMPLE
		Answers: 1. scissors 2. clock 3. cat Problem: Color each arrow pointing right blue and each arrow pointing left red.  Answer:  TEACHER NOTES: Location is an important aspect of describing a spatial situation. To identify an object, we often need to indicate where it is; for example, we may say, “the rectangle or the desk is to the left of the table.” These positional descriptions sometimes involve measurements—for example, “the rectangle is one inch below the circle”—or relations—for example, “it is outside the circle but inside the square/at the top of the square on the right.”

CONCEPT	EXPECTATION	EXAMPLE
		Visualization refers to a student's ability to construct a single image or a series of related images, whether the image is of an object or a set of directions. Sometimes, visualization involves imaging what an object looks like from a different point of view, for example, from above rather than from the side. Other times, it involves predicting what an object might look like after it is turned or flipped.[2]

[2] National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (pp. 31, 67). Reston, VA: Author.

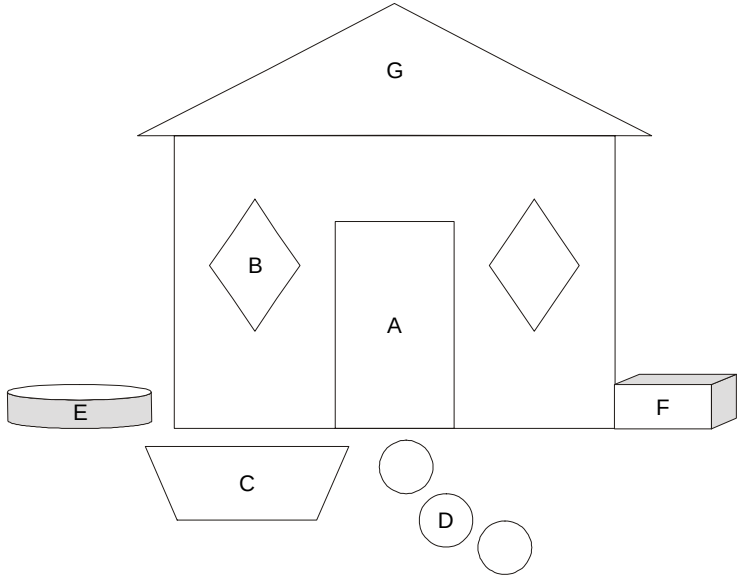
BIG IDEA (3): Apply transformations and use symmetry to analyze mathematical situations

	CONCEPT	EXPECTATION	EXAMPLE
A	Use transformations on objects	Use manipulatives to model slides and turns	Students choose a basic shape (ideally one that is not regular with equal angles and equal sides) and using that shape create a design by sliding and/or turning the shape. Example:  slide flip TEACHER NOTES: Students should learn about the flip (a mirror image of the original shape) since it is the most basic transformation (change in position), and other transformations can be obtained by combining flips. Additionally, a shape resulting from a slide (orienting the shape the same way but sliding it to a different position) should also be explored at this level.[3]

DEFINITION:
transformation—the mapping or moving of all points of a figure in a plane according to a common operation.[4]

[3] National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (p. 49). Reston, VA: Author.
[4] *Math at hand*: *A mathematics handbook* (p. 536). (1999). Wilmington, MA: Great Source Education Group, Inc.

BIG IDEA (4): Use visualization, spatial reasoning and geometric modeling to solve problems

	CONCEPT	EXPECTATION	EXAMPLE
A	Recognize and draw three-dimensional represent-ations	Recognize geometric shapes and structures in the student's environment and specify the shape's location	Teachers can find many opportunities and resources for encouraging students to observe shapes in their environment. For example, they can take children on a shape hunt through the school, the playground, or in the nearby neighborhood. Problem: What shape is each lettered part of the house and items in the lawn below? 

CONCEPT	EXPECTATION	EXAMPLE
		Answer: A. rectangle B. rhombus C. trapezoid D. circle E. cylinder F. rectangular prism G. triangle TEACHER NOTES: Spatial visualization can be developed by building and manipulating first concrete then mental representations of shapes. To foster spatial visualization and reasoning, teachers can ask students to visualize the path they just walked from the library and describe it by specifying landmarks along the route or to talk about how solid shapes look from different perspectives.[5]

[5] National Council of Teachers of Mathematics. (2001). *Principles and standards for school mathematics* (pp. 100–101). Reston, VA: Author.

GEOMETRIC AND SPATIAL RELATIONSHIPS

Grade 2

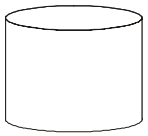
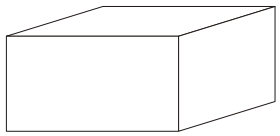
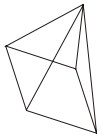
BIG IDEA (1): Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

	CONCEPT	EXPECTATION	EXAMPLE
A	Describe and use geometric relationships	Describe attributes and parts of 2- and 3-dimensional shapes (circle, triangle, trapezoid, rectangle, rhombus, sphere, rectangular prism, cylinder, pyramid)	Students should have a number of opportunities to describe shapes and figures by their attributes. As students and teachers discuss shapes and figures within their environment, teachers should help students recognize the mathematical language used to describe the attributes or parts of 2- and 3-dimensional shapes/figures. Give pairs of students a polyhedron model (rectangular prism, cylinder, or pyramid) along with some toothpicks and thawed frozen peas. Demonstrate to the students how to create a "skeleton" of the model they have been given, using the toothpicks and peas. After they have completed their skeleton model, have them answer the following questions about their models. • How many faces? • How many corners? • How many edges?

DEFINITION:
attributes—characteristics such as shape, size, and color.[1]

parts—faces, vertices, sides, edges, lines, etc.[2]

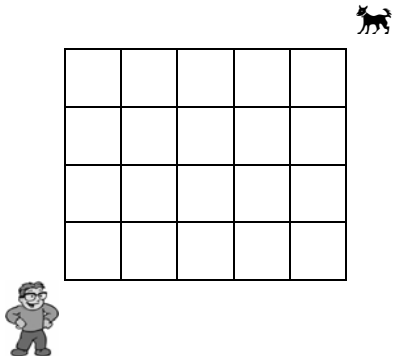
[1] Eather, J. *A math dictionary for kids*. Retrieved June 5, 2004, from http://www.amathdictionaryforkids.com.
[2] National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (p. 2). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE
		After all groups have answered the questions about their models, have them sort the models by characteristics such as • the number of edges • the shapes of the faces • the number of corners Teachers may want to take geometric shapes or figures and carefully describe their parts using the appropriate mathematical vocabulary. For instance, they might describe the faces of a rectangular prism, pointing to the vertices and edges of the prism. They then ask students to do the same—choose a shape or figure and describe its parts. Problem: Which of the shapes below 1. has all rectangular faces? 2. has a circle base? 3. has 8 vertices (corners)? 4. has triangles for faces?    A B C

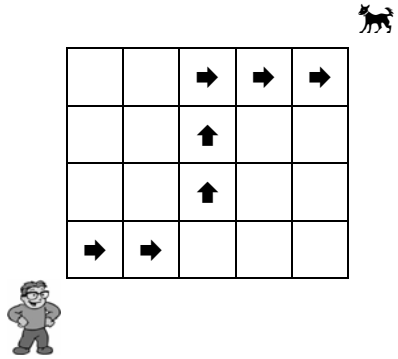
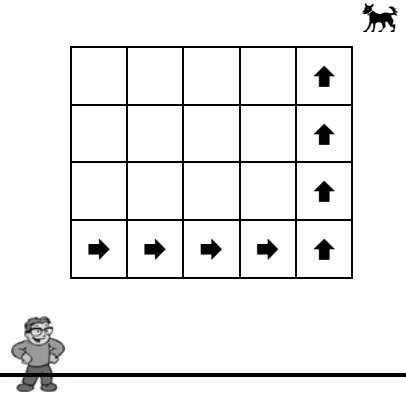
CONCEPT	EXPECTATION	EXAMPLE
		Answers: 1. B 2. A 3. B 4. C TEACHER NOTES: Students should be beginning to use the appropriate mathematical language for two- and three-dimensional shapes. Student should recognize the shapes in a set of pattern blocks, describing the blue and tan shapes as either rhombi (rhombus) or parallelograms. Many students and adults will want to call these shapes diamonds, but "diamond" is not a mathematical term. "Naming shapes is important, but even more important is observing the attributes, or characteristics, of shapes. Characteristics of two-dimensional shapes that children might explore include • the number of sides and vertices (corners), and • how shapes can be put together or taken apart to make other shapes." Characteristics of three-dimensional figures that children might focus on include the number and shape of the faces, the number of edges and corners, places where the figure is wide or narrow, and what the figure would look like if it were cut apart and opened up.[3]

[3] National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (pp. 9–11). Reston, VA: Author.

BIG IDEA (2): Specify locations and describe spatial relationships using coordinate geometry and other representational systems

	CONCEPT	EXPECTATION	EXAMPLE
A	Use coordinate systems	Find and name locations with simple relationships on a map (coordinate system)	Students could create and interpret maps of familiar places. Divide students into small groups, and provide each group with a sheet of chart paper and a set of blocks to create a map of the classroom. Tell each group to show a path on its map to get from point A to point B in the room, e.g., from the reading corner to the door.[4] Ask them questions such as "How many different paths did you find? Is there a shortest path?" Teachers might also take a look at the NCTM Illuminations Web site, illuminations.nctm.org, for the Logo activity "Can You Get the Turtle to the Pond?" The investigation allows students to map the paths that a turtle travels. Problem: Have students trace different paths from the boy to the dog, using the positional words that you read to them. 

[4] National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (p. 42). Reston, VA: Author.

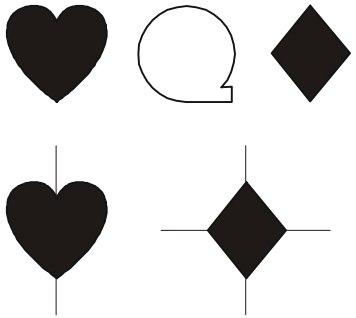
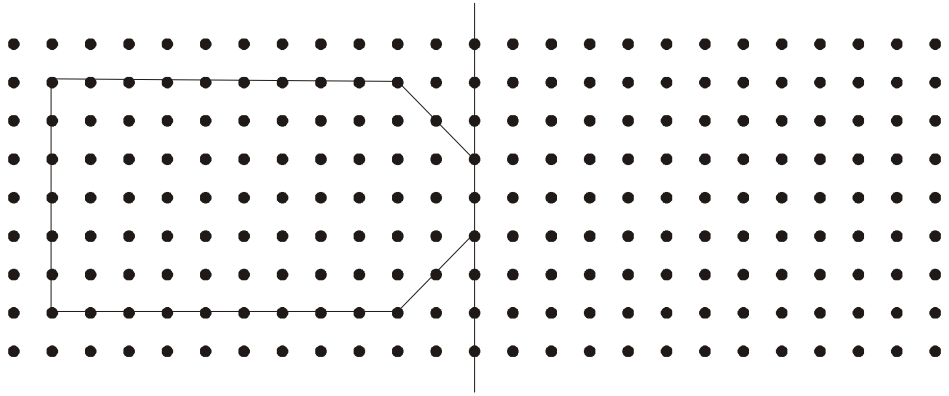
CONCEPT	EXPECTATION	EXAMPLE
		Answer: *Answers may vary. Examples:*  

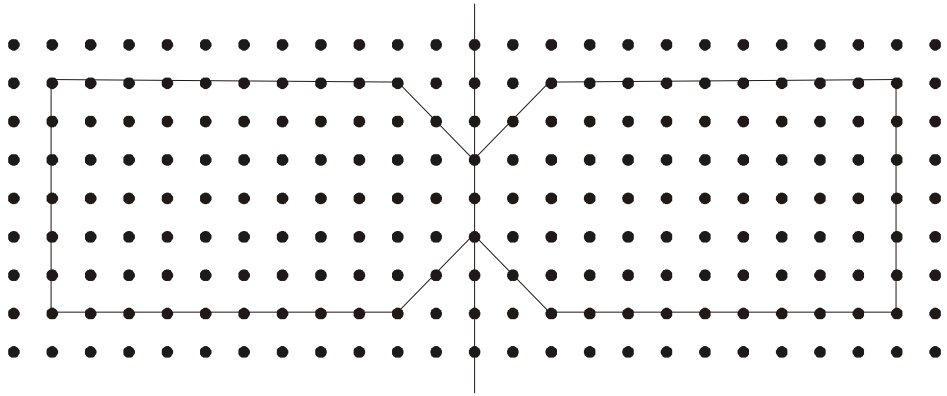
CONCEPT	EXPECTATION	EXAMPLE
		TEACHER NOTES: By the end of grade 2, students should be able to use positional vocabulary to describe and interpret relative positions, including the terms near, far, backward, forward, inside, outside, above, below, up, down, over and under. They should be able to create and interpret simple maps and name locations on simple coordinate grids.

BIG IDEA (3): Apply transformations and use symmetry to analyze mathematical situations

	CONCEPT	EXPECTATION	EXAMPLE
A	Use transformations on objects	Use manipulatives to model flips	Students choose a basic shape (ideally one that is not a regular shape with equal angles and equal sides), then use that shape to create a design by flipping the shape. Have students draw the original shape first and then the design they've created after flipping the shape. Students may also want to use slides and turns for their designs. TEACHER NOTES: Students learn about transformations because they are a basic tool for defining equivalence of shape; two shapes are congruent if one shape can be transformed into another by a simple slide, flip or turn or by a combination of these transformations.[5]
C	Use symmetry	Recognize and create shapes that have symmetry	Problem: Color each shape below that has a line of symmetry. Then cut each shape that you colored and fold on the line of symmetry. 

[5] National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (p. 49). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE
		Answer:  Problem: Using dot paper, complete the symmetrical shape. 

CONCEPT	EXPECTATION	EXAMPLE
		Answer:  Students explore lines of symmetry of familiar shapes with mirrors or Miras and through paper folding. Give each student a paper square. Direct the students to fold the square in half, open the papers, and point to the fold lines. Tell the students, "Another name for the fold line is the line of symmetry. One side of the line is a copy (or mirror image) of what is on the other side of the line." To help the students verify your explanation, place a mirror or Mira on each student's fold line, and have the student check if one side is a mirror image of the other side. Have them use crayons to color their fold lines. Encourage students to find other lines of symmetry on their squares, and have them use a crayon of a different color for each line.[6]

[6] National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (p. 57). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE
		TEACHER NOTES: "Teachers should guide students to recognize, describe, and informally prove the symmetric characteristics of designs through the materials they [teachers] supply and the questions they ask. Students can use pattern blocks to create designs with line and rotational symmetry or use paper cutouts, paper folding, and mirrors to investigate lines of symmetry." [7]

[7] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 100). Reston, VA: Author.

BIG IDEA (4): Use visualization, spatial reasoning and geometric modeling to solve problems

	CONCEPT	EXPECTATION	EXAMPLE
A	Recognize and draw three-dimensional represen-tations	Recognize and represent shapes from different perspectives	Problem:  Which shape below is the same size and shape as the one above?  A B C Answer: B (*Have students cut out the original shape to see which shape it fits over to verify their answer*.) Flash (show for a short time) a transparency of a design made from basic shapes. Designs can range from simple to complex but all should be made with basic shapes. Students can describe (orally, in writing, or through illustration) what they saw and how they saw it.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Match each lettered shape below with the correct name for the shape.  1. rectangle 2. triangle 3. trapezoid Answer: 1. B 2. A, D 3. C

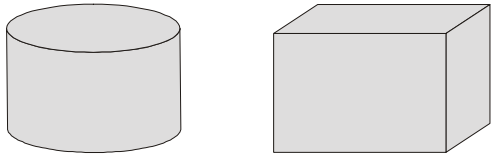
CONCEPT	EXPECTATION	EXAMPLE
		TEACHER NOTES: Visualization refers to a student's ability to construct a single image or a series of related images, whether the image is of an object or a set of directions. Sometimes visualization involves imaging what an object looks like from a point of view, for example, from above rather than from the side. Other times, it involves predicting what an object might look like after it is turned or flipped or what the shadow of an object might look like. Visual memory—ability to recall what a shape looked like or what has been changed about a shape or an arrangement—is prerequisite to the ability to manipulate mental images, an important part of spatial sense.[8]

[8] National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (p. 67). Reston, VA: Author.

GEOMETRIC AND SPATIAL RELATIONSHIPS

Grade 3

BIG IDEA (1): Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

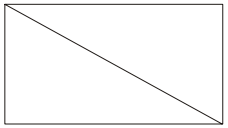
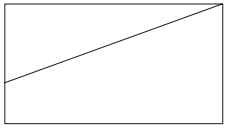
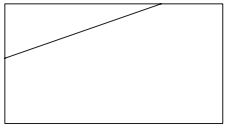
	CONCEPT	EXPECTATION	EXAMPLE
A	Describe and use geometric relationships	Compare 2- and 3-dimensional shapes by describing their attributes (circles, rectangle, rhombus, trapezoid, triangle, rectangle, prism, cylinder, pyramid and sphere)	Problem:  A B Match the following statements to the shapes above. 1. I have curved edges. 2. My faces are rectangles and squares. 3. I have circular faces. 4. I have only straight edges.

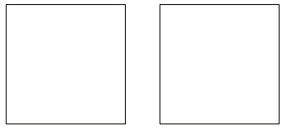
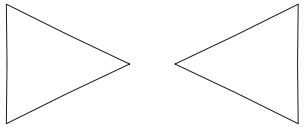
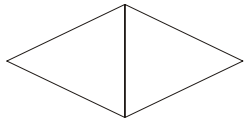
DEFINITION:
attributes—a characteristic or distinctive feature—such as shape, size, color—of an object or given set of objects.[1]

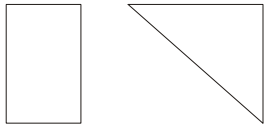
[1] Eather, J. *A math dictionary for kids*. Retrieved June 5, 2004, from http://www.amathdictionaryforkids.com.

CONCEPT	EXPECTATION	EXAMPLE
		Answers: 1. A 2. B 3. A 4. B Students work in pairs for this activity, with partners facing each other or sitting back to back. If the partners sit face-to-face, put a divider between them so the two cannot see each other's work. First, one student creates a design. Then he/she gives directions to the partner so the partner can re-create the design using blocks, without looking at it. The re-creator then explains to the designer what the design looks like. Finally, they remove the divider and compare their designs to see if the designs match. Students will switch roles and repeat the activity. A variation would be the teacher giving directions to the entire class and pairs of students trying to recreate the design. *Note*: All students need the same set of blocks. When describing your design, be sure to include the terms outlined in the grade-level expectation (e.g., circle, rectangle) as well as positional words such as above, below, left, right, perpendicular.[2] Give students various 2- and 3-dimensional shapes and have them sort the shapes into different groups while giving their reasons for sorting.

[2] Gavin, M. K., Belkin, L., Spinelli, A. M., St. Marie, J. (2001). *Navigating through geometry in grades 3–5* (pp. 11–14). Reston, VA: National Council of Teachers of Mathematics.

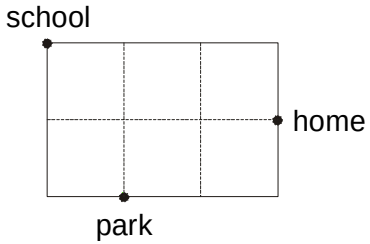
	CONCEPT	EXPECTATION	EXAMPLE
C	Compose and decompose shapes	Predict the results of putting together or taking apart 2- and 3-dimensional shapes.	PROBLEM: If you cut along the lines in the three rectangles below, which rectangle would become a triangle and a trapezoid? 1.  2.  3.  Answer: 2

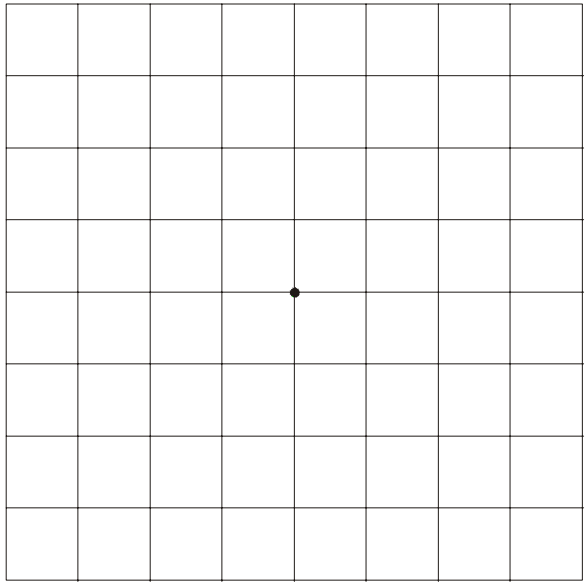
CONCEPT	EXPECTATION	EXAMPLE
		Problem: What shape do you get if the two shapes below are pushed together to have one common side?  Answer: Rectangle Problem: Show how Alfonzo could combine the two shapes below to make a rhombus.  Answer: 

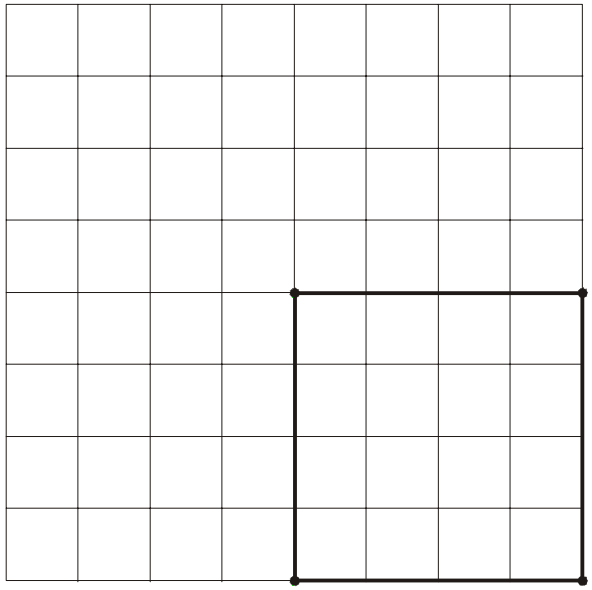
CONCEPT	EXPECTATION	EXAMPLE
		Problem: If the following two shapes were combined, what new shape would be created?  Answer: Pentagon TEACHER NOTES: Give students opportunities for subdividing various shapes. Give them opportunities also for combining shapes to create new ones and drawing models of their combined shapes.[3]

[3] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 165–166). Reston, VA: Author.

BIG IDEA (2): Specify locations and describe spatial relationships using coordinate geometry and other representational systems

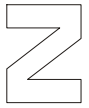
	CONCEPT	EXPECTATION	EXAMPLE
A	Use coordinate systems	Describe location using common language and geometric vocabulary (forward, back, left, right, north, south, east, west)	Problem: 1. Use the map below to find out where John is now if he started at home, walked one block north then three blocks west. 2. Give another path John could take from home to reach the same place.  Answers: 1. School 2. He starts at home, goes two blocks west, one block north, and one block west.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: From the center of the grid, go 4 units right, and mark the point; go 4 units down, and mark the point; go 4 units left, and mark the point; go 4 units up, and mark the point. Connect the points in the order that you drew them. What shape do you get? 

CONCEPT	EXPECTATION	EXAMPLE
		Answer:  A square TEACHER NOTES: Using grids and developing fundamental ideas and strategies for navigating them are important components of discrete mathematics.[4]

[4] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 165–168). Reston, VA: Author.

BIG IDEA (3): Apply transformations and use symmetry to analyze mathematical situations

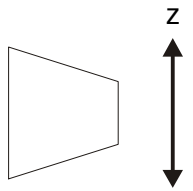
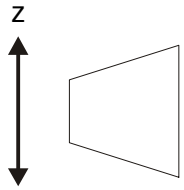
	CONCEPT	EXPECTATION	EXAMPLE
A	Use transform-ations on objects	Determine if two objects are congruent through a slide, flip or turn	Problem:  Which of the three figures below is congruent to the figure above? 1.  2.  3.  Answer: 2

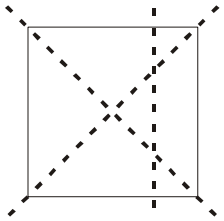
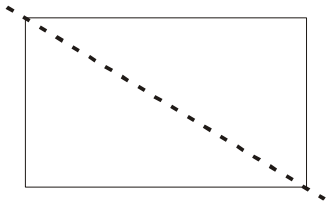
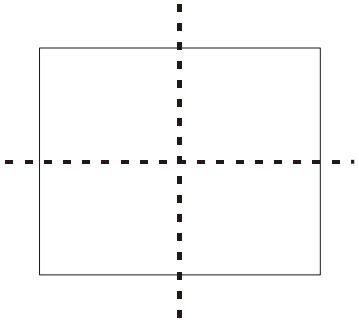
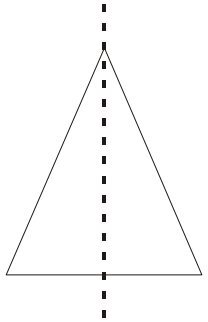
DEFINITIONS:

congruent—Objects that have the same shape and size are congruent.[5]

transformations—the mapping or moving of all points of a figure in a plane according to a common operation.[6]

[5] Abdelnoor, J. R. E. (1979). *The Silver Burdett mathematical dictionary* (Rev. Ed.) (p. 21). Silver Burdett Press: Morristown, New Jersey.
[6] *Math at hand*: *A mathematics handbook* (p. 536). (1999). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Draw a flip of the figure below over line z.  Answer: 

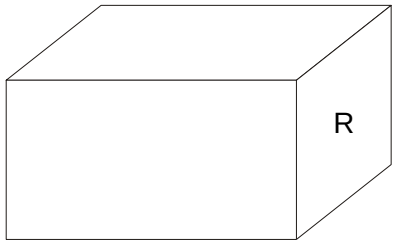
CONCEPT	EXPECTATION	EXAMPLE
C Use symmetry	Identify lines of symmetry in polygons	Problem: Which of the following shapes have all lines of symmetry drawn? 1.  2.  3.  4.  Answer: 3 and 4 TEACHER NOTES: A line of symmetry divides a figure into two congruent halves that are mirror images of each other.[7]

[7] Cavanagh, M. (2000). *Math to know* (p. 455). Wilmington, MA: Great Source Education Group, Inc.

GEOMETRIC AND SPATIAL RELATIONSHIPS

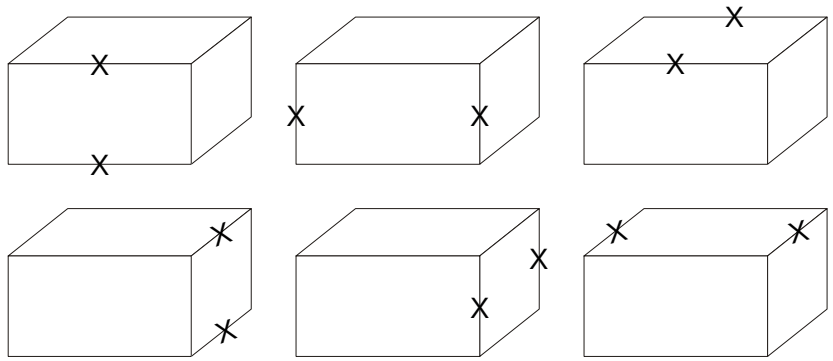
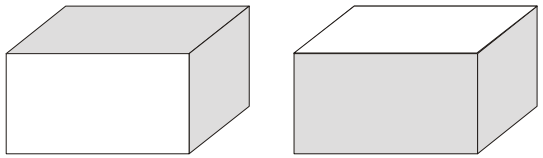
Grade 4

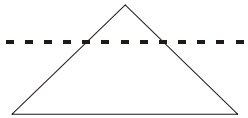
BIG IDEA (1): Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

	CONCEPT	EXPECTATION	EXAMPLE
A	Describe and use geometric relationships	Identify and describe the attributes of 2- and 3-dimensional shapes (prisms, cones, parallelism, perpendicularity)	Problem: In the rectangular prism below, all intersecting edges and adjacent sides are perpendicular. 1. What is the shape of the face opposite side R that you cannot see? 2. Put an X on two parallel lines on the rectangular prism. 3. Shade two perpendicular faces the same color. 

DEFINITION:
attributes—a characteristic or distinctive feature—such as shape, size, color—of an object or given set of objects.[1]

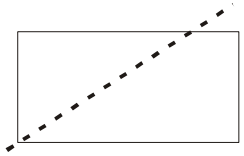
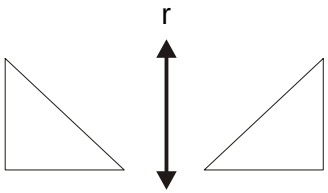
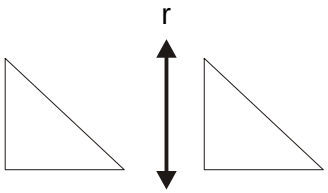
[1] Eather, J. *A math dictionary for kids*. Retrieved June 5, 2004, from http://www.amathdictionaryforkids.com.

CONCEPT	EXPECTATION	EXAMPLE
		Answers: 1. Rectangle 2.  3. *Answers may vary. Examples:* 

	CONCEPT	EXPECTATION	EXAMPLE
C	Compose and decompose shapes	Describe the results of subdividing, combining and transforming shapes	Problem: If you cut along the line in the shape below, you will have two shapes. What are the new shapes?  Answer: Triangle and trapezoid. Problem: Draw one line across the rectangle to create a trapezoid and triangle. 

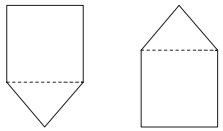
DEFINITION:
transforming shapes—changing plane figures by mapping or moving every point to a new location.[2]

[2] *Math at hand*: *A mathematics handbook* (p. 536). (1999). Wilmington, MA: Great Source Education Group, Inc.

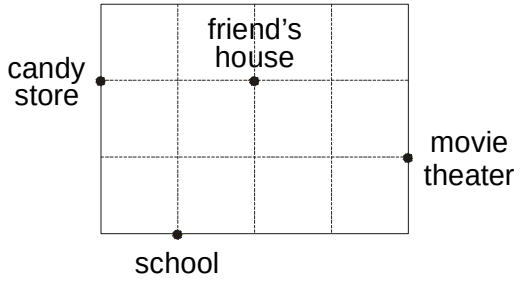
CONCEPT	EXPECTATION	EXAMPLE
		Answer:  Problem: Which of the following pictures is an example of a reflection (flip) over line r? 1.  2.  Answer: 1

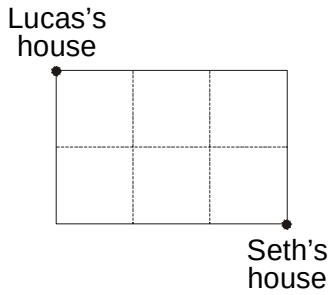
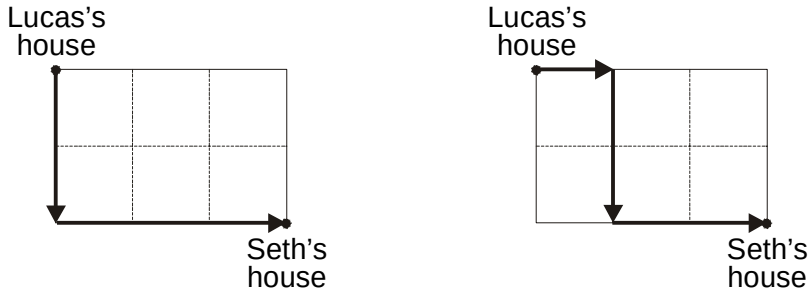
DEFINITION:
reflection (flip)—a transformation creating a mirror image of a figure on the opposite side of a line.[3]

[3] *Math at hand*: *A mathematics handbook* (p. 533). (1999). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Combine a triangle and a square along congruent sides to make a pentagon. Answer: *Answers may vary. Examples:* 

BIG IDEA (2): Specify locations and describe spatial relationships using coordinate geometry and other representational systems

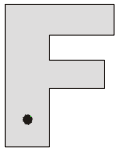
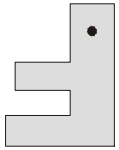
	CONCEPT	EXPECTATION	EXAMPLE
A	Use coordinate systems	Describe movement using common language and geometric vocabulary (forward, back, left, right, north, south, east, west)	Problem: 1. Use the following grid and directions to see where Cathy went after she left the candy store: Cathy walked two blocks east, one block south, then two blocks east again. Where did she end up? 2. Using the same grid, describe a path that Cathy could take from school to the movie theater.  Answer: 1. Cathy went to the movie theater. 2. Three blocks east and one block north or one block north and three blocks east.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Lucas wants to walk to Seth's house. Describe a path he could take to get there.  Answer: *Answers may vary. Two possibilities include the following:*  Lucas walks two blocks south, then three blocks east; or he walks one block east, two blocks south, then two blocks east.

CONCEPT	EXPECTATION	EXAMPLE
		TEACHER NOTES: Using grids and developing fundamental ideas and strategies for navigating them are important components of discrete mathematics.[4]

[4] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 165–168). Reston, VA: Author.

BIG IDEA (3): Apply transformations and use symmetry to analyze mathematical situations

	CONCEPT	EXPECTATION	EXAMPLE
A	Use transformations on objects	Predict the results of sliding/ translating, flipping/ reflecting or turning/rotating around the center point of a polygon	Problem: What will the following shape look like if it is turned (rotated) clockwise180° around the point shown?  Answer: 

DEFINITIONS:

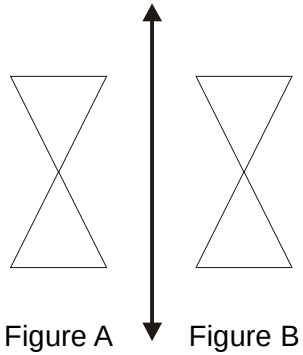
flipping/reflecting—a transformation creating a mirror image of a figure on the opposite side of a line.[5]

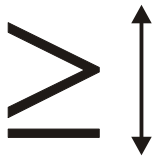
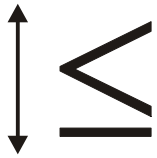
sliding/translating—a transformation that involves sliding a figure a given distance in a given direction.[6]

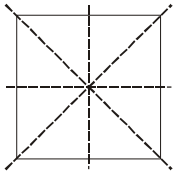
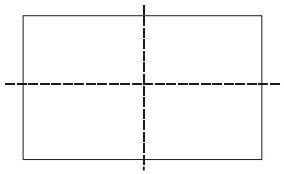
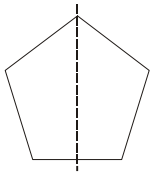
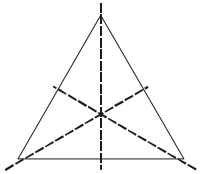
transformations—the mapping, or movement, of all points of a figure in a plane according to a common operation. Examples of the operation include rotations, dilations, reflections, and translations.[7]

turning/rotating—a transformation that involves turning a figure at a given angle and in a given direction around a point.[8]

[5] *Math at hand*: *A mathematics handbook* (p. 533). (1999). Wilmington, MA: Great Source Education Group, Inc.
[6] *Math at hand*: *A mathematics handbook* (p. 536). (1999). Wilmington, MA: Great Source Education Group, Inc.
[7] Intermath Dictionary. Retrieved August 10, 2005 from www.intermath-uga.gatech.edu/dictnary.
[8] *Math at hand*: *A mathematics handbook* (p. 534). (1999). Wilmington, MA: Great Source Education Group, Inc.

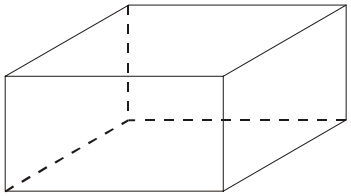
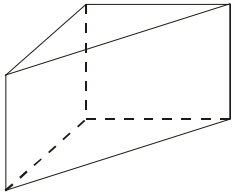
CONCEPT	EXPECTATION	EXAMPLE
		Problem: Joe and Jill are discussing which transformation of Figure A created Figure B (see below). Joe says it was a slide (translation), and Jill says it was flip (reflection) across the line. Who is correct?  Figure A Figure B Answer: They are both correct. Figure B could have been created by either a slide (translation) or flip (reflection) of Figure A.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Draw a flip of the following symbol across the line.  Answer: 

CONCEPT	EXPECTATION	EXAMPLE
C Use symmetry	Construct a figure with multiple lines of symmetry and identify the lines of symmetry.	Problem: Draw each of the shapes named below and all the lines of symmetry in each. 1. a square 2. a rectangle 3. a regular pentagon 4. an equilateral triangle Answer: 1.  2.  3.  4.  TEACHER NOTES: A line of symmetry divides a figure into two congruent halves that are mirror images of each other.[9]

[9] Cavanagh, M. (2000). *Math to know* (p. 455). Wilmington, MA: Great Source Education Group, Inc.

BIG IDEA (4): Use visualization, spatial reasoning and geometric modeling to solve problems

CONCEPT	EXPECTATION	EXAMPLE
Recognize and draw three-dimensional representations	Given the picture of a prism, identify the shapes of the faces	Problem: Identify the shapes of the faces of each prism. 1.  2.  Answers: 1. rectangles 2. rectangles and triangles

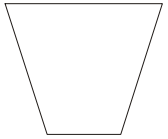
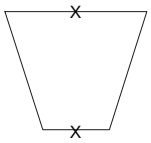
DEFINITION:
prism—A 3-dimensional figure in which all of the surfaces are polygons.[10]

[10] Kaplan, A. (1998). *Math on call* (p. 588). Wilmington, MA: Great Source Education Group, Inc.

GEOMETRIC AND SPATIAL RELATIONSHIPS

Grade 5

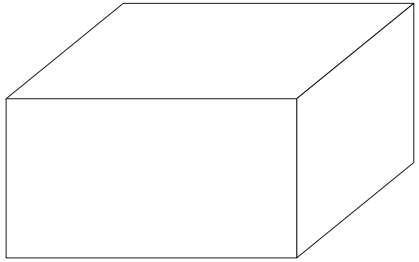
BIG IDEA (1): Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

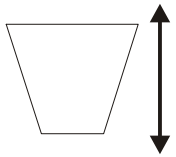
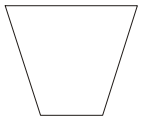
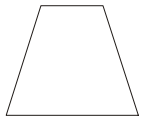
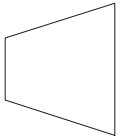
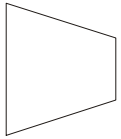
	CONCEPT	EXPECTATION	EXAMPLE
A	Describe and use geometric relationships	Analyze 2- and 3-dimensional shapes by describing the attributes	Problem: Put an X on the parallel lines in the trapezoid below, and explain why the shape is a trapezoid.  Answer:  This shape is a trapezoid because it has four sides and one pair of the opposite sides are parallel.

DEFINITIONS:

attributes—a characteristic or distinctive feature—such as shape, size, color—of an object or given set of objects.[1]

[1] Eather, J. *A math dictionary for kids*. Retrieved June 5, 2004, from http://www.amathdictionaryforkids.com.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Draw a rectangular prism, then determine how many pairs of parallel edges it has. Answer:  A rectangular prism has 12 pairs of parallel edges: 2 pairs on each of the six faces of the prism.

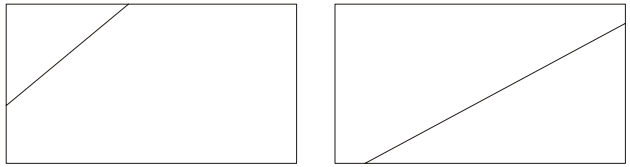
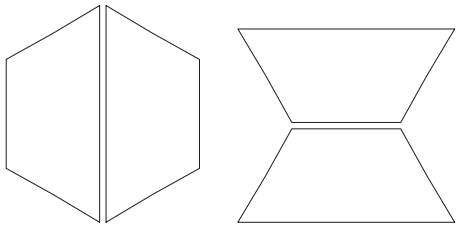
	CONCEPT	EXPECTATION	EXAMPLE
C	Compose and decompose shapes	Predict and justify the results of subdividing, combining and transforming shapes	Problem: If a translation is performed on the shape below,  which of the following shapes will it look like? 1.  3.  2.  4.  Answer: 1

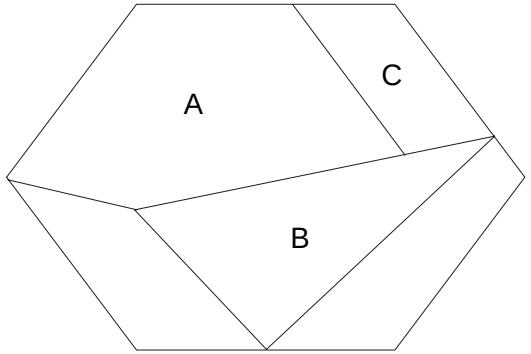
DEFINITION:

transforming shapes—Changing plane figures by moving every point in a plane figure to a new location.[2]

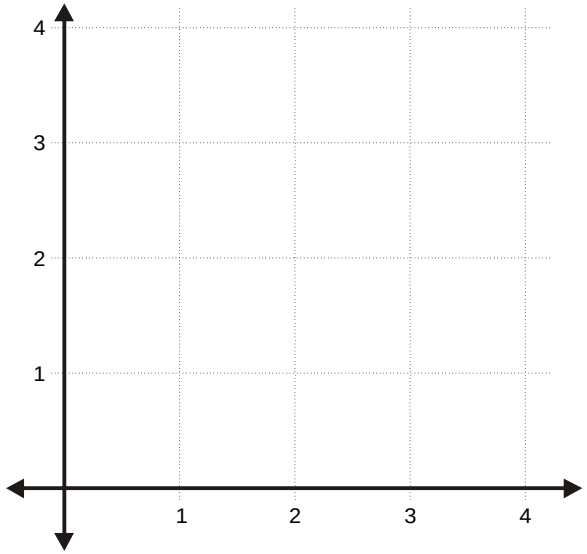
translation—a transformation in which a figure is slid a given distance in a given direction.[3]

[2] *Math at hand*: *A mathematics handbook* (p. 536). (1999). Wilmington, MA: Great Source Education Group, Inc.
[3] *Math at hand*: *A mathematics handbook* (p. 536). (1999). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Draw one line segment across the shape below to make a triangle and a pentagon.  Answer: *Answers may vary. Examples:*  Problem: Draw two shapes that can be put together to form a hexagon. Answer: *Answers may vary. Examples:* 

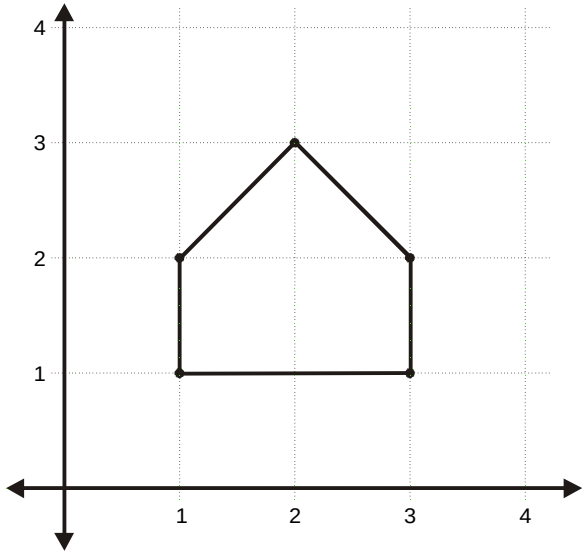
CONCEPT	EXPECTATION	EXAMPLE
		Problem: Identify each shape that has a letter on it in the diagram below.  Answers: A is a pentagon. B is a triangle. C is a trapezoid.

BIG IDEA (2): Specify locations and describe spatial relationships using coordinate geometry and other representational systems

	CONCEPT	EXPECTATION	EXAMPLE
A	Use coordinate systems	Use coordinate systems to specify locations, describe paths and find the distance between points along horizontal and vertical lines	Problem: 1. Using the grid below, plot the following points: (1,1) (1,2) (2,3) (3,2) (3,1). Then connect the points in the same order to draw the figure. 2. What is the distance from point (1,2) to point (3,2)? 

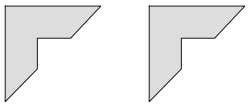
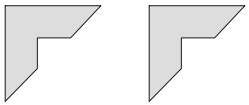
DEFINITION:
coordinate systems—two-dimensional systems in which the coordinates of a point are its distances from two intersecting, usually perpendicular straight lines called axes.[5]

[5] Cavanagh, M. (2000). *Math to know* (p. 446). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		Answers: 1.  2. 2 units TEACHER NOTES Using grids and developing fundamental ideas and strategies for navigating them are important components of discrete mathematics.[6]

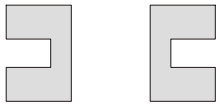
[6] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 165–168). Reston, VA: Author.

BIG IDEA (3): Apply transformations and use symmetry to analyze mathematical situations

	CONCEPT	EXPECTATION	EXAMPLE
A	Use transform-ations on objects	Predict, draw and describe the results of sliding/ translating, flipping/ reflecting and turning/rotating around a center point of a polygon	Problem: In each example below, a transformation has been performed on the first shape to create the second shape. Decide if the statement that describes the transformation in each example is true or false. Shape 1 Shape 2 1.  A slide (translation) across a line created the second figure. 2.  A turn (rotation) of 180° created the second figure.

DEFINITIONS:
flipping/reflecting—a transformation creating a mirror image of a figure on the opposite side of a line.[7]
sliding/translating—a transformation that involves sliding a figure a given distance in a given direction.[8]
transformation—the mapping, or movement, of all points of a figure in a plane according to a common operation. Examples of the operation include rotations, dilations, reflections, and translations.[9]
turning/rotating—a transformation that involves turning a figure at a given angle and in a given direction around a point.[10]

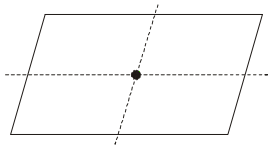
[7] *Math at hand*: *A mathematics handbook* (p. 533). (1999). Wilmington, MA: Great Source Education Group, Inc.
[8] *Math at hand*: *A mathematics handbook* (p. 536). (1999). Wilmington, MA: Great Source Education Group, Inc.
[9] Intermath Dictionary. Retrieved August 10, 2005 from www.intermath-uga.gatech.edu/dictnary.
[10] *Math at hand*: A Mathematics handbook (p. 534). (1999). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		Answers: 1. True 2. False Problem: Draw a shape and apply either a slide (translation), turn (rotation), or flip (reflection). Draw a picture of the new shape and identify the transformation you used to create it. Answer: *Answers may vary. Example:* Shape 1 Shape 2 Description of the transformation  A turn (rotation) to the right created the second shape.  A flip (reflection) to the right created the second shape.

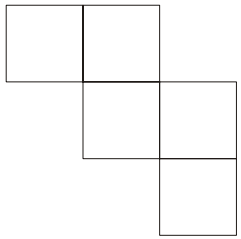
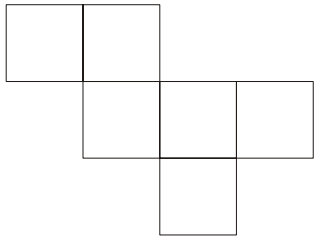
CONCEPT	EXPECTATION	EXAMPLE
C Use symmetry	Identify polygons and designs with rotational symmetry	Problem: Which figure below does not have rotational symmetry?  1. an equilateral triangle 2. a square 3. a pentagon 4. a circle Answer: 3

DEFINITION:
rotational symmetry—a property that enables a figure to be mapped onto itself by a rotation of 180° or less.[11]

[11] *Geometry to go* (p. 470). (2001). Wilmington, MA: Great Source Education Group, Inc.

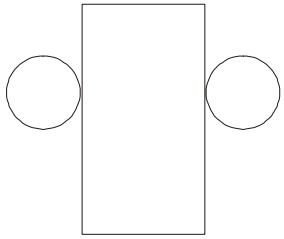
CONCEPT	EXPECTATION	EXAMPLE
		Problem: Place a point in the blank parallelogram below to show that this shape has rotational symmetry.  Answer:  (*If students have difficulty placing the point, have them trace the first figure and use it to mark the second figure.*) TEACHER NOTES: To determine if a figure has rotational symmetry, trace it then fold to establish the center. Hold you tracing paper over the original figure with your pencil at its center then see if the tracing paper maps over the original figure in a half-turn (180°) or less.

BIG IDEA (4): Use visualization, spatial reasoning and geometric modeling to solve problems

	CONCEPT	EXPECTATION	EXAMPLE
A	Recognize and draw three-dimensional represent-ations	Given a net of a prism or cylinder, identify the 3-dimensional shape	Problem: Complete the net of the cube, a rectangular prism with all square faces, by drawing the missing face.  Answer: 

DEFINITION:
net of a prism—a flat 2-dimensional shape that can be folded into a 3-dimensional solid.[12]

[12] Eather, J. *A math dictionary for kids*. Retrieved June 5, 2004, from http://www.amathdictionaryforkids.com.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: What solid figure can you make by folding the pattern below?  1. pentagon 2. prism 3. cylinder 4. tetrahedron Answer: 3

GEOMETRIC AND SPATIAL RELATIONSHIPS

Grade 6

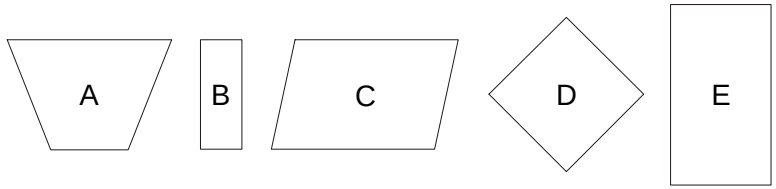
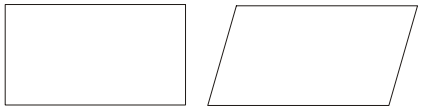
BIG IDEA (1): Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

CONCEPT	EXPECTATION	EXAMPLE
A Describe and use geometric relationships	Identify the properties of 1- 2- and 3- dimensional shapes using the appropriate geometric vocabulary.	Problem: Read each statement below and decide if it is always true, sometimes true, or never true. 1. Parallelograms are squares. 2. All squares are rectangles. 3. Rhombuses are squares. 4. All rectangles are squares. Answers: 1. Sometimes true 2. Always true 3. Sometimes true 4. Never true

DEFINITIONS:

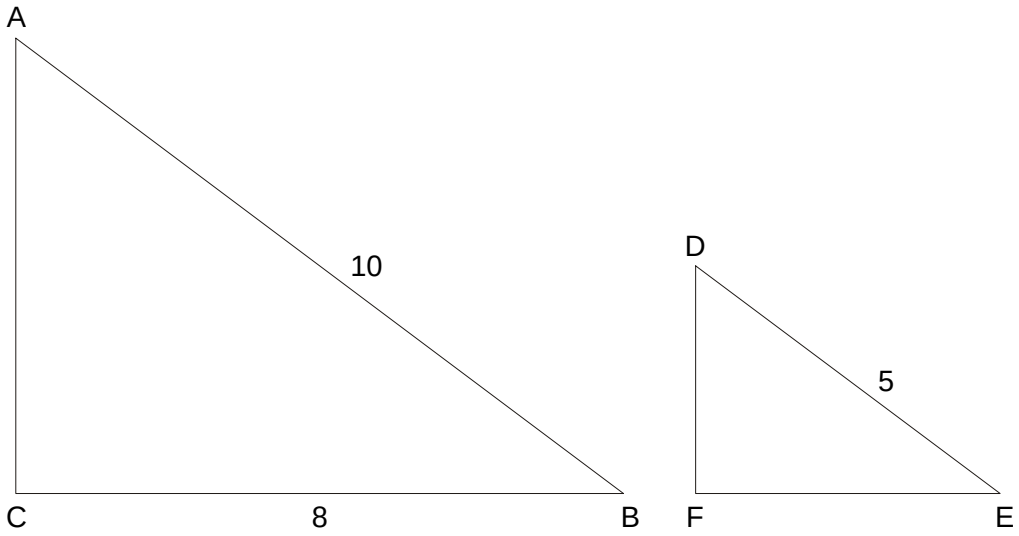
properties of 1-2- and 3- dimensional shapes—common features of 1-, 2-, and 3-dimensional shapes, such as number and length of sides, angle measures, etc.[1]

[1] Eather, J. *A math dictionary for kids*. Retrieved August 25, 2004, from http://www.amathdictionaryforkids.com.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: 1. Which of the following shapes are rectangles? Why? 2. Which of the following shapes are parallelograms? Why? 3. Which of the following shapes are rhombi? Why?  Answers: 1. B, D, and E are rectangles since all the angles in these shapes are right angles and the opposite sides are parallel and congruent. 2. B, C, D, and E are parallelograms since the opposite sides in the shapes are parallel and congruent. 3. D is a rhombus because all four sides are congruent. Problem: Draw an example of a parallelogram that is NOT a rhombus. Answer: *Answers may vary. Examples:* 

CONCEPT	EXPECTATION	EXAMPLE
		TEACHER NOTES: “Students must carefully examine the features [properties] of shapes in order to precisely define and describe fundamental shapes.” They might be asked to make and record measurements of the sides and angles to observe features and characteristics of certain shapes. Students may also be asked to draw the diagonals of multiple examples of shapes to determine defining characteristics of shapes. “Investigations into the properties of, and relationships among, similar shapes can afford students many opportunities to develop and evaluate conjectures inductively and deductively.” [2]

[2] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 233–234). Reston, VA: Author.

	CONCEPT	EXPECTATION	EXAMPLE
B	Apply geometric relationships	Describe relationships between the corresponding angles and the length of corresponding sides of similar triangles (whole number scale factors)	Problem: Right triangle ABC is similar to right triangle DEF. If AB = 10, BC = 8, and DE = 5, find the measure of side EF and the measure of angle EFD. 

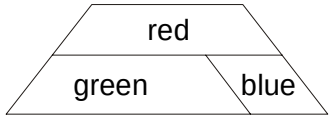
DEFINITIONS:

corresponding angles—angles that are in the same relative position in similar or congruent figures.[3]

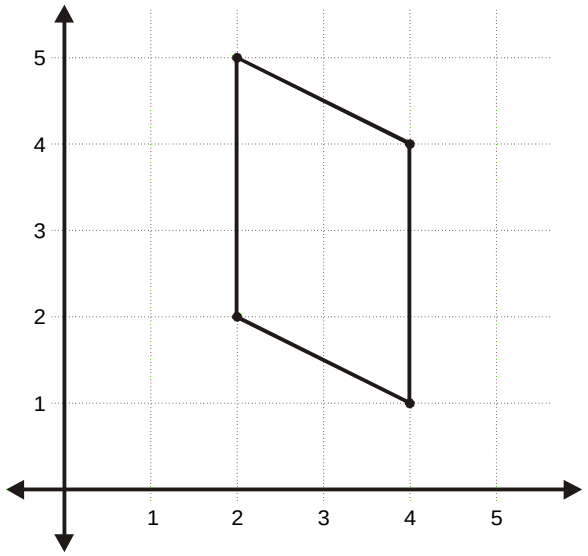
corresponding sides of similar triangles—Sides that are in the same relative position in similar or congruent figures. Similar triangles are triangles that have proportional corresponding sides and congruent corresponding angles.[4]

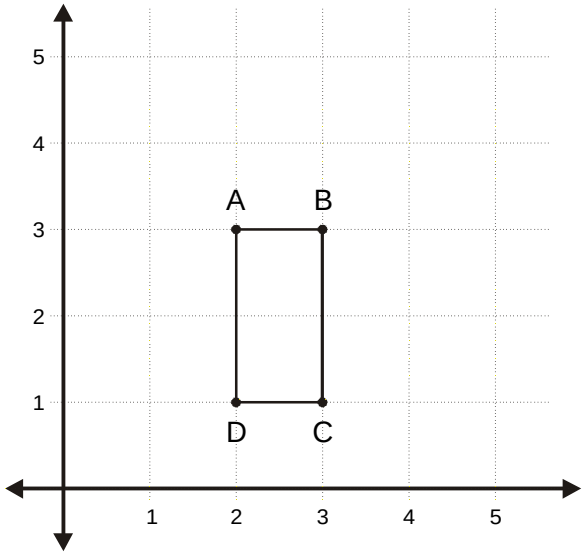
[3] *Geometry to go* (p. 452). (2001). Wilmington, MA: Great Source Education Group, Inc.
[4] *Geometry to go* (pp. 452, 472). (2001). Wilmington, MA: Great Source Education Group, Inc.

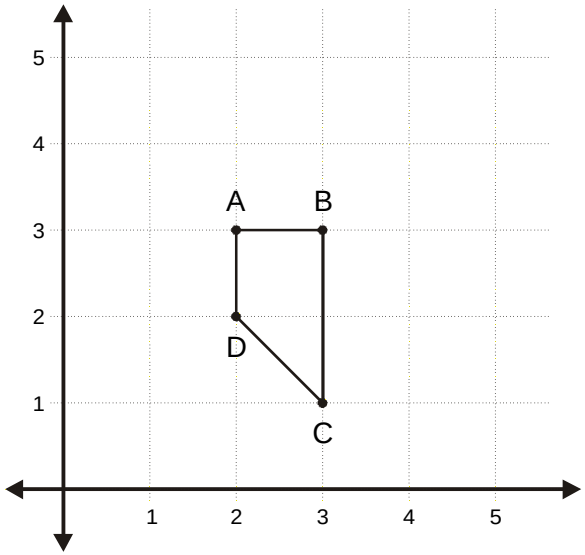
CONCEPT	EXPECTATION	EXAMPLE
		Answer: $\frac{AB}{BC} = \frac{DE}{EF}$, and angle EFD is a 90 degree angle. $\frac{10}{8} = \frac{5}{x}$ *Some students might use cross products to solve:* 10 = 40 x = 4 FE = 4 *Some students might use equivalent fractions to solve:* $\frac{10 \div 2}{8 \div 2} = \frac{5}{4}$ FE = 4

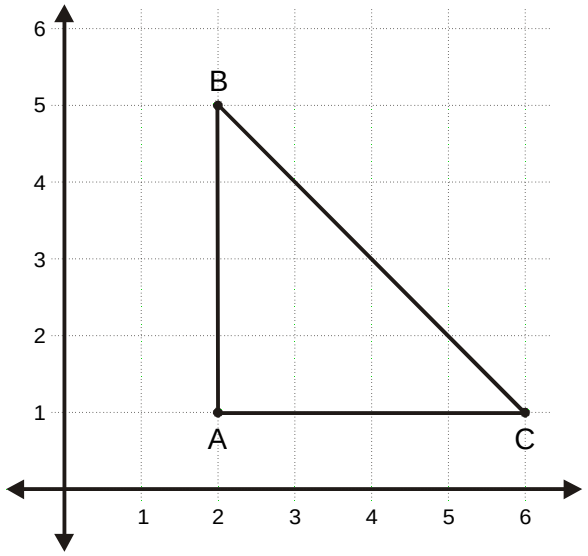
CONCEPT	EXPECTATION	EXAMPLE
		Problem: Use pattern blocks to build the figure below. Is the large trapezoid formed similar to the green trapezoid? Explain how you know.  Answer: The large trapezoid is not similar to the green trapezoid, since the non-parallel sides of the large trapezoid are twice as long as the sides of the green one, but the longer base on the bottom is only 1½ times as long as the original, and the smaller base on top is equal to the original. Problem: In a drawing of a Missouri map, the distance across, which is actually 240 miles, is represented by 4 inches. Find the scale factor that is used, and then determine the distance on the map going from north to south if the actual distance is 420 miles. Answer: Scale factor = 240 miles / 4 inches = 60 miles per inch 420 miles / 60 miles per inch = 7 inches So the distance for 420 miles in the drawn map is 7 inches.

BIG IDEA (2): Specify locations and describe spatial relationships using coordinate geometry and other representational systems

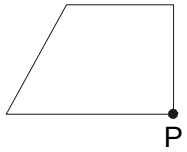
	CONCEPT	EXPECTATION	EXAMPLE
A	Use coordinate systems	Use coordinate geometry to construct geometric shapes	Problem: Graph the following points on a coordinate axis, and connect the points in the order given: (2,2) (2,5) (4,4) (4,1) What shape is made? Answer:  The shape is a parallelogram.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Plot the following coordinates—A (2,3) B (3,3) C (3,1)—and determine the coordinates of D so that 1. ABCD is a rectangle 2. ABCD is a trapezoid Answers: 1.  The coordinates of D are (2,1).

CONCEPT	EXPECTATION	EXAMPLE
		2.  The coordinates of D could be (2,2), (2,0) or (1,1).

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Plot the following points, and connect them in the order given: A (2,1) B (2,5) C (6,1) and A (2,1). What shape do you get? Explain how you know. Answer:  *Answers may vary. However, all explanations should mention that the triangle has a right angle and two equal sides. Example:* AB and AC are equal sides because each is four units long. Angle A is a right angle, which makes triangle ABC an isosceles right triangle.

BIG IDEA (3): Apply transformations and use symmetry to analyze mathematical situations

	CONCEPT	EXPECTATION	EXAMPLE
A	Use transform-ations on objects	Describe the transformation from a given pre-image to its image using the terms reflection/flips, rotation/turn and translation/slide	Problem: Draw a 90° clockwise rotation of this trapezoid around point P. 

DEFINITIONS:

image—a figure that is created after a shape undergoes a transformation.[5]

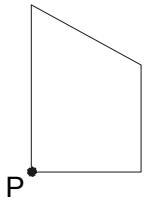
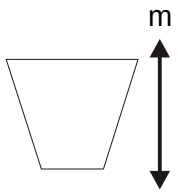
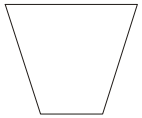
pre-image—the original figure in a transformation.[6]

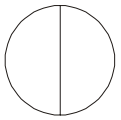
reflection/flips—a transformation in which a figure is flipped over a line called the line of reflection. All corresponding points in the image and pre-image are equidistant from the line of reflection.[7]

rotation/turn—a transformation that forms an image by turning its pre-image about a point.[8]

translation/slide—a transformation in which an image is formed by moving every point on a figure the same distance in the same direction. Points in the original figure are equidistant from their image.[9]

[5] *Geometry to go* (p. 467). (2001). Wilmington, MA: Great Source Education Group, Inc.
[6] *Geometry to go* (p. 458). (2001). Wilmington, MA: Great Source Education Group, Inc.
[7] *Geometry to go* (p. 469). (2001). Wilmington, MA: Great Source Education Group, Inc.
[8] *Geometry to go* (p. 470). (2001). Wilmington, MA: Great Source Education Group, Inc.
[9] *Geometry to go* (p. 475). (2001). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		Answer:  Problem: 1. Draw the reflection of the figure below across line m. 2. Is it possible to get the same image by translating (sliding) or rotating the figure?  Answers: 1.  2. You can get the same image by translating (sliding) across line L, but not by rotating.

CONCEPT	EXPECTATION	EXAMPLE
C Use symmetry	Identify polygons and designs with rotational symmetry	Problem: 1. Create a hubcap design that has $\frac{1}{2}$ turn (180^{o}) rotational symmetry. 2. Draw a polygon that has $\frac{1}{3}$ turn (120°) rotational symmetry. *Answers: Answers may vary. Examples:* 1.  2. 

DEFINITION:
rotational symmetry—a property that allows a figure to be mapped (look the same) as it is rotated 180 degrees or less.[9]

[9] *Geometry to go* (p. 470). (2001). Wilmington, MA: Great Source Education Group, Inc.

BIG IDEA (4): Use visualization, spatial reasoning and geometric modeling to solve problems

	CONCEPT	EXPECTATION	EXAMPLE
A	Recognize and draw three-dimensional represent-ations	Use spatial visualization to identify isometric representations of mat plans	Problem: Which mat plan does the isometric drawing on the next page represent? A. 3 4 / 1 3 B. 4 3 / 1 5 C 1 3 / 4 1 D 3 3 / 1 4

DEFINITIONS:

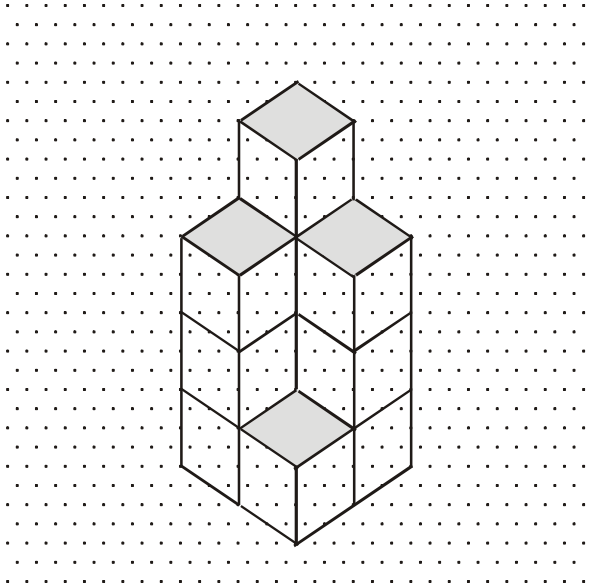
isometric representations—drawings that provide a corner view of an object, thus showing three dimensions.[10]

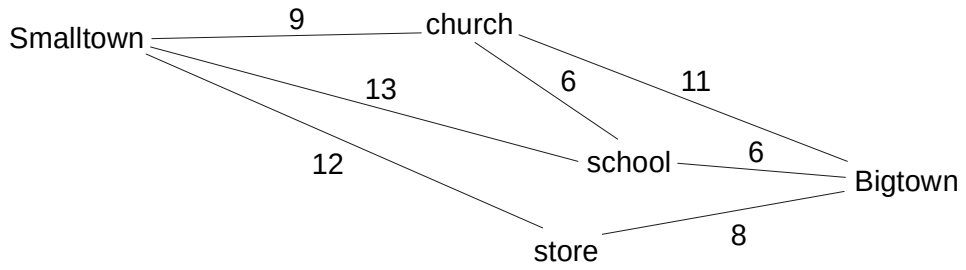
mat plans—drawings of the base of a cube, with numbers on the squares to show how high each stack of cubes is.[11]

3	4
1	3

[10] *Geometry to go* (p. 459). (2001). Wilmington, MA: Great Source Education Group, Inc.

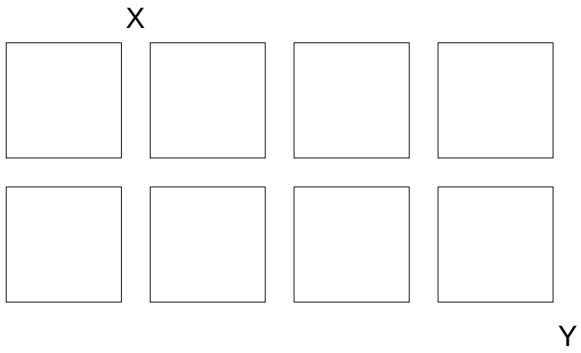
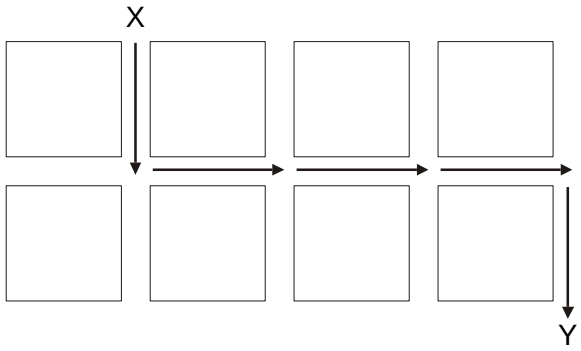
[11] Lappan, G. Frey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2002). Ruins of Montarek spatial visualization. *Connected mathematics* (p. 9). Glenview, IL: Prentice Hall.

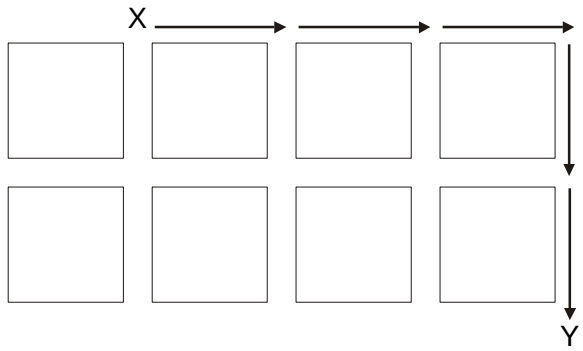
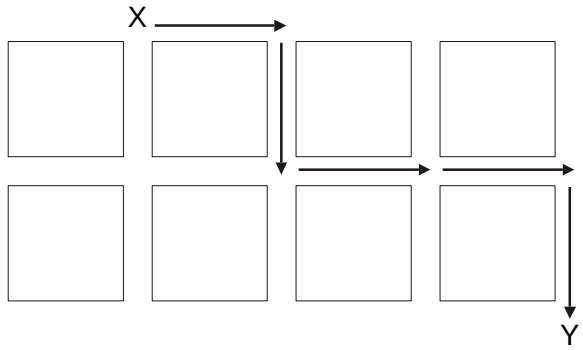
CONCEPT	EXPECTATION	EXAMPLE
		 Answer: A TEACHER NOTES: For lessons and more information on spatial reasoning using cubes and isometric drawings, go to the NCTM Web site, http://illuminations.nctm.org.

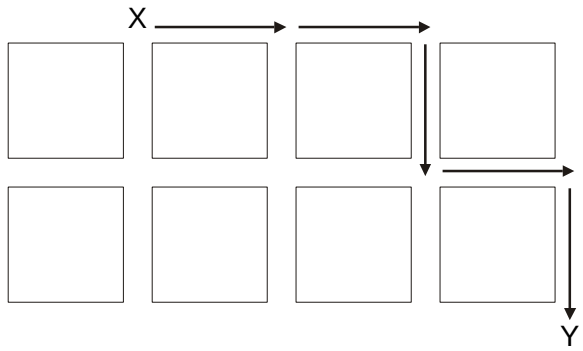
	CONCEPT	EXPECTATION	EXAMPLE
B	Draw and use visual models	Draw or use visual models to represent and solve problems	Problem: John needs to go from Smalltown to Bigtown, as represented in the figure below. If he wants to go the shortest route, should he go past the church, the school, the store, or both the church and school (The numbers in the figure below indicate the distance (in miles) from each spot)?  Answer: He should go past the school because the distance would be the shortest—19 miles.

DEFINITION:
visual model—models such as networks that could be used to analyze and solve real problems such as those concerned with efficiency. The models of 2- and 3-dimensional objects may also assist in the students' reasoning about spatial relationships.[12]

[12] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 237). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: In the figure below, find all possible routes that carriage horses could take to get from the stables (X) to the entrances of Scenic Park (Y).  Answer: 

CONCEPT	EXPECTATION	EXAMPLE
		or  or 

CONCEPT	EXPECTATION	EXAMPLE
		or  or